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GEOMETRY OF TREFOIL CONE – MANIFOLD*

By
D. DEREVNIN, A. MEDNYKH, AND M. MULAZZANI
(Received March 8, 2011)

Abstract. In this paper we prove that Trefoil knot cone manifold $\mathcal{T}(\alpha)$ with cone angle α is spherical for $\pi/3 < \alpha < 5\pi/3$. We show also that its spherical volume is given by the formula $\mathrm{Vol}(\mathcal{T}(\alpha)) = (3\alpha - \pi)^2/12$.

1. Introduction

Let $\mathcal{T}(\alpha)$ be a cone manifold whose underlying space is the three-dimensional sphere $\mathcal{S}^3$ and singular set is Trefoil knot $\mathcal{T}$ with cone angle α (Fig. 1). Since $\mathcal{T}$ is a toric knot by the Thurston theorem its complement $\mathcal{T}(0) = \mathcal{S}^3 \setminus \mathcal{T}$ in the $\mathcal{S}^3$ does not admit hyperbolic structure. We think this is the reason why the simplest nontrivial knot came out of attention of geometricians. However, it is well known that Trefoil knot admits geometric structure. H. Seifert and C. Weber (1935) [16] have shown that the spherical space of dodecahedron (= Poincaré homology 3-sphere) is a cyclic 5-fold covering of $\mathcal{S}^3$ branched over $\mathcal{T}$. Topological structure and fundamental groups of cyclic n-fold coverings have described by D. Rolfsen [14] and A.J. Sieradsky [18]. In spite of positive solution of the Orbifold Geometrization Conjecture given in [1] and [2] the geometrical structure of $\mathcal{T}(\alpha)$ for an arbitrary α is still unknown. The most progress is achieved for the case $\alpha = 2\pi/n$, $n \in \mathbb{N}$. In that case $\mathcal{T}(2\pi/n)$ is a geometric orbifold, that is can be represented in the form $\mathbb{X}^3/\Gamma$, where $\mathbb{X}^3$ is one of the eight three-dimensional homogeneous geometries and Γ is a discrete group of isometries of $\mathbb{X}^3$. By Dunbar [4] classification of non-hyperbolic orbifolds has a spherical structure for $n \leq 5$, Nil for $n = 6$ and $\widetilde{\mathrm{PSL}}(2, \mathbb{R})$ for $n \geq 7$. Quite

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ON SOME NEW POSITIONAL SMALL INDUCTIVE DIMENSIONS FOR UNIFORM SPACES*

By
D. N. GEORGIOU

(Received March 23, 2011
Revised April 4, 2012)

Abstract. The paper defines new positional dimension-like functions of the type ind for uniform spaces and presents several theorem concerning the standard properties of dimension theory for these functions. Finally, some open questions concerning these functions are given.

1. Preliminaries

It was observed in the book of Gillman-Jerison (see [9]) that a better dimension theory can be built out, for covering dimension, if we do not consider all open sets, but only that base of them, that consist of the cozero sets (i.e., where a continuous function is not 0). Then many statements, originally valid for normal spaces, extend to all Tychonoff spaces. Later it was realized, by Charalambous, that the same idea can be extended much further: for all uniform spaces one can define covering dimension by (uniform) cozero sets (i.e., where a uniformly continuous function is not 0). Of course, this theory of dimension depends only on the system of cozero sets, not on the actual uniformity. Nevertheless, the usual setting is that of uniform spaces, these theorems are considered to belong to the theory of uniform spaces.

The paper intends to contribute to this theory. Its setting is a pair of uniform spaces, one a subspace of the other one, for which there are defined two basic types of small inductive dimension-like functions, and several theorems are proved for them. The paper follows rather closely the presentation of the paper [8] who investigated the corresponding theorems for topological spaces.

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ON PROPERTIES OF GENERALIZED NEIGHBOURHOOD SYSTEMS

By
ERDAL GUNER AND SEVDA SAGIROGLU

(Received July 1, 2011)

Abstract. Neighbourhood structures are particular cases of generalized neighbourhood systems. Let $X \neq \emptyset$ be a set and $N(X)$ be the set of all neighbourhood structures on X, partially ordered as follows: $\psi \leq \varphi$ for $\psi, \varphi \in N(X)$ iff $\psi(x) \leq \varphi(x)$ for each $x \in X$. Then $N(X)$ is a complete sublattice of the $GN(X)$ which denotes the set of all strongly generalized neighbourhood systems on X partially ordered as above. We investigate some properties of $GN(X)$. In addition we discuss the product of generalized neighbourhood systems and present some new results concerning *gn*-continuity related to this product.

1. Introduction

In 1914, Hausdorff [6] defined toplogical spaces in terms of a system of neighbourhoods at each point. Csaszar [1] continued to study this approach under the name of neighbourhood spaces, with various conditions on the systems of neighbourhoods at each point. Recently, the properties of neighbourhood spaces have investigated by using neighbourhood p-stacks instead of neighbourhood filters in [7,9] and Richmond and Slapal [11] continued to study these concepts by using neighbourhood rasters which is a subclass of neighbourhood p-stacks. The concept of generalized neighbourhood systems which is a strict generalization of neighbourhood structures recalled below was given by Csaszar [2]. Let $X \neq \emptyset$ then a map $\psi\colon X \to \exp(\exp X)$ satisfying $x \in V$ for $V \in \psi(x)$, $x \in X$ is called a generalized neighbourhood system (briefly GNS) on X. In this paper, $GN(X)$ denotes the set of all strongly generalized neighbourhood structures on X partially ordered as follows: $\psi \leq \varphi$ for $\psi, \varphi \in GN(X)$ iff $\psi(x) \subseteq \varphi(x)$ for

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GENERALIZED ABSOLUTE CONVERGENCE OF FOURIER SERIES

By
R. G. VYAS

(Received June 1, 2012)

Abstract. Here, sufficiency conditions are obtain for the convergence of the Fourier series of the form $\sum_{k\in Z}\varpi_{|k|}(\varphi(|\hat{f}(n_k)|))$, where $\hat{f}(n_k)$ are Fourier coefficients of f, $\{\varpi_n\}$ is a certain sequence of positive numbers, $\varphi(u)$ $(u \geq 0)$ is an increasing concave function and $\{n_k\}_{k=1}^{\infty}$ is an increasing sequence of natural numbers with $n_{-k} = -n_k$ for all k.

1. Introduction

Let f be a 2π-periodic real function in $L^1[0, 2\pi]$ and

$$f(x) \sim \frac{1}{2}a_0 + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx) \equiv \sum_{k\in Z}\hat{f}(k)e^{ikx}, \tag{1}$$

be the Fourier series of f, wherein a_n, b_n are Fourier coefficients of f and $\hat{f}(k) = \frac{a_{|k|} - ib_{|k|}sgn(k)}{2}$, $(k \in \mathbb{Z})$.

Generalizing the concept of β-absolute convergence of Fourier series [4], for $f \in L^p([0, 2\pi])$ $(1 < p \leq 2)$ L. Leindler [2] obtained sufficiency condition for the convergence of the series

$$\sum_{k\in Z}\varpi_{|k|}(\varphi(|\hat{f}(k)|)), \tag{2}$$

where $\varphi(u)$ $(u \geq 0, \varphi(0) = 0)$ is an increasing and concave function, $\varpi_0 = 0$ and $\{\varpi_n\}_{n=1}^{\infty}$ is a certain sequence of positive numbers. For $\varpi_n = n^0$, $\forall n$ and $\varphi(x) = x^{\beta}$ $(0 < \beta \leq 1)$, one gets β-absolute convergence of Fourier series.

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ON GENERALIZED α-CLOSED SETS IN ISOTONIC SPACES

By
M. PARIMALA AND S. JAFARI

(Received June 14, 2012)

Abstract. The purpose of the present paper is to introduce the concept of generalized α-closed sets in isotonic spaces and study their fundamental properties. The generalized closed sets are then used to define generalized α-continuous functions and investigate some of their characterizations.

1. Introduction

Closure spaces and (more generally) isotonic spaces have already been studied by Hausdorff [8], Day [1], Hammer [6,7], Gnilka [2,3,4], Stadler [10, 11], and Habil and Elzenati [5].

A function μ from the power set $P(X)$ of a nonempty set X into itself is called a generalized closure operator (briefly GCO) on X and the pair (X, μ) is said to be generalized closure space (briefly GCS). Generalized closure spaces, a strong generalization of topological spaces, have application in several branches of pure and applied mathematics, as lattice theory, logic, general topology, digital topology and convex geometry. In 1993 Maki et al [9] introduced the notion of generalized α-closed sets in topology. For each result known in topological spaces it is interesting to find out which are the minimal assumption that allow its extension to generalized closure spaces.

As in topological spaces, there are many hereditary properties that hold in isotonic spaces, and we note that not every property which holds in topological spaces must hold in isotonic spaces. However, since every topological space is an isotonic space, we note that if a property does not hold in a topological space, it must not hold in any isotonic space either. In this paper, we introduced the notion of generalized α-closed sets in (X, μ) and study some of its basic properties.

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ON MIXED QUASI-EINSTEIN MANIFOLDS

By
SAHANOUS MALLICK AND UDAY CHAND DE
(Received August 28, 2012)

Abstract. The object of the present paper is to introduce a new type of Riemannian manifold called mixed quasi-Einstein manifolds $M(QE)_n$ and prove the existence Theorem of a mixed quasi-Einstein manifold. Some geometric properties of mixed quasi-Einstein manifolds have been studied. The totally umbilical hypersurfaces of $M(QE)_n$ are also studied. The existence of a mixed quasi-Einstein manifold have been proved by two non-trivial examples.

1. Introduction

A Riemannian manifold (M^n, g), $n = \dim M \geq 2$, is said to be an Einstein manifold if the following condition

$$S = \frac{r}{n} g \tag{1}$$

holds on M, where S and r denote the Ricci tensor and the scalar curvature of (M^n, g) respectively. According to ([1],p.432), (1) is called the Einstein metric condition. Einstein manifolds play an important role in Riemannian Geometry as well as in general theory of relativity. Also Einstein manifolds form a natural subclass of various classes of Riemannian manifolds by a curvature condition impossed on their Ricci tensor ([1],p.432–433). For instance, every Einstein manifold belongs to the class of Riemannian manifolds (M^n, g) realizing the following relation:

$$S(X, Y) = ag(X, Y) + bA(X)A(Y), \tag{2}$$

where a, $b \in \mathbb{R}$ and A is a non-zero 1-form such that

$$g(X, U) = A(X), \tag{3}$$

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A LIE ALGEBRA APPROACH TO DIFFERENCE SETS: HOMAGE TO YAHYA OULD HAMIDOUNE

By
GYULA KÁROLYI

(Received March 19, 2013)

Abstract. We demonstrate how the adjoint representation of the general linear Lie algebra over a finite dimensional vector space may be used in the study of difference sets. This approach extends quite naturally to a purely matrix algebraic proof of the Cauchy–Davenport theorem given previously in the language of tensor algebra by Dias da Silva and Hamidoune.

1. Introduction

Given an abelian group $G \neq 0$, let $p(G)$ denote the smallest possible order of a nontrivial subgroup in G. In case $G = \mathbb{F}^+$ is the additive group of a field $\mathbb{F}$, we simply write $p(\mathbb{F})$. Thus, $p(\mathbb{F})$ equals the characteristic of the field $\mathbb{F}$ if it is positive, otherwise $p(\mathbb{F}) = \infty$.

For subsets $\mathcal{A}, \mathcal{B} \subseteq G$, their sumset is defined as

$$\mathcal{A} + \mathcal{B} := \{a + b \mid a \in \mathcal{A}, b \in \mathcal{B}\}.$$

In the special case when $\mathcal{B} = -\mathcal{A} := \{-a \mid a \in \mathcal{A}\}$ we simply write $\mathcal{A} - \mathcal{A}$ instead of $\mathcal{A} + (-\mathcal{A})$. A classical result of Cauchy [2] and Davenport [3] can be phrased as follows.

Theorem 1. *Let $\mathcal{A}, \mathcal{B}$ be nonempty subsets of an abelian group G. Then*

$$|\mathcal{A} + \mathcal{B}| \geq \min\{|\mathcal{A}| + |\mathcal{B}| - 1, p(G)\}.$$

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ITERATE (i,j)-m-STRUCTURES AND ITERATE (i,j)-m-CONTINUITY

By
TAKASHI NOIRI AND VALERIU POPA

(Received April 17, 2013)

Abstract. We introduce the notion of $(i,j)mIT$-open sets determined by operators m_X^i-Int and m_X^i-Cl ($i = 1,2$) on a bi-m-space (X, m_X^1, m_X^2). By using $(i,j)mIT$-open sets, we introduce and investigate a function $f\colon (X, m_X^1, m_X^2) \to (Y, \sigma_1, \sigma_2)$ called $(i,j)mIT$-continuous. As a special case of $(i,j)mIT$-continuous functions, we obtain (i,j)-m-precontinuous functions due to Carpintero et al. [7].

1. Introduction

The concepts of minimal structures (briefly m-structures) and minimal spaces (briefly m-spaces) are introduced by the present authors in [27] and [28]. In these papers, they introduced M-continuous functions and m-continuous functions and obtained their basic properties. Moreover, in [21] and [24], they extended the study of continuity between bitopological spaces to the study of m-continuity and M-continuity beteen minimal stuructures. Quite recently, in [14]-[18], Min and Kim introduced the notions of m-semi-open sets, m-preopen sets, m-α-open sets and m-β-open sets which are generalizations of semi-open sets, preopen sets, α-open sets and β-open sets, respectively. And also, they introduced the notions of m-semi-continuity, m-precontinuity, m-α-continuity and m-β-continuity which are generalizations of the notions of semi-continuity, precontinuity, α-continuity and β-continuity, respectively. In [6], [33] and [34], the notions of m-semi-open sets, m-preopen sets, m-α-open sets and m-β-open sets are also introduced and studied. In [26], the present authors introduced the notions of iterate minimal structures and iterate m-continuity.

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MIXED CONNECTEDNESS IN GTS VIA HEREDITARY CLASSES

By
SHYAMAPADA MODAK AND TAKASHI NOIRI

(Received July 26, 2013)

Abstract. We introduce six forms of connected sets on a generalized topological space with a hereditary class and investigate their relations and also their unified properties.

1. Introduction

The notion of ideal topological spaces was studied by Kuratowski [9] and Vaidyanathswamy [14]. The notion was further investigated by Janković and Hamlett [7]. Recently, the notion of $*$-connected ideal topological spaces has been introduced and studied in [6, 13, 10].

Császár [5] introduced the notion of a generalized topological space with hereditary class. This is a generalization of an ideal topological space. In this paper, we introduce six forms of connected sets on a generalized topological space with a hereditary class and investigate their relations and also their unified properties.

2. Preliminaries

Let X be a nonempty set and $P(X)$ the power set of X. A subset μ of $P(X)$ is called a generalized topology (GT) [1, 2, 3] if $\emptyset \in \mu$ and the arbitrary union of members of μ is in μ. A generalized topology μ is called a quasi-topology [4] on X if $U,\ V \in \mu$ implies $U \cap V \in \mu$. A nonempty subset $\mathcal{H}$ of $P(X)$ is called a hereditary class [5] of X if $A \subset B,\ B \in \mathcal{H}$ implies $A \in \mathcal{H}$. For each subset A of X, a subset $A^*(\mathcal{H})$ (briefly A^*) of X is defined in [5] as follows: $A^*(\mathcal{H}) = \{x \in \in X\colon U \cap A \notin \mathcal{H}$ for every $U \in \mu$ containing $x\}$. If $c_{\mu^*}(A) = A \cup A^*$ for each

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ON WEAKLY-Q-SYMMETRIC MANIFOLDS

By
PRAJJWAL PAL, AND U. C. DE

(Received August 6, 2013)

Abstract. The object of the present paper is to study weakly-Q-symmetric manifolds $(WQS)_n$. At first some geometric properties of $(WQS)_n$ $(n > 2)$ have been studied. Next we consider the decomposability of $(WQS)_n$. Finally, we give two examples of the $(WQS)_4$.

1. Introduction

As is well known, symmetric spaces play an important role in differential geometry. The study of Riemannian symmetric spaces was initiated in the late twenties by Cartan [4], who, in particular, obtained a classification of those spaces. Let (M^n, g), $(n = \dim M)$ be a Riemannian manifold, i.e., a manifold M with the Riemannian metric g, and let ∇ be the Levi-Civita connection of (M^n, g). A Riemannian manifold is called locally symmetric [4] if $\nabla R = 0$, where R is the Riemannian curvature tensor of (M^n, g). This condition of local symmetry is equivalent to the fact that at every point $P \in M$, the local geodesic symmetry $F(P)$ is an isometry [18] . The class of Riemannian symmetric manifolds is very natural generalization of the class of manifolds of constant curvature. During the last six decades the notion of locally symmetric manifolds have been weakened by many authors in several ways to different extent such as conformally symmetric manifolds by Chaki and Gupta [5], recurrent manifolds introduced by Walker [25], conformally recurrent manifolds by Adati and Miyazawa [1], pseudo symmetric manifolds by Chaki [6], weakly symmetric manifolds by Tamássy and Binh [23] etc.

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SUFFICIENT CONDITIONS FOR SUPRA β-CONTINUITY

By
IVAN KUPKA

(Received September 27, 2013)

Abstract. We give two sufficient conditions for functions to be supra β-continuous. A notion of a new generalized derivative is introduced. The methods presented here can be used also for other kinds of generalized continuity.

1. Introduction

In this paper we give two sufficient conditions for functions to be supra β-continuous. One of the ways how we can see, that an object (e.g. a function) has some nice property is to compare it with another object with the same property. We do this kind of a comparison more often than we think. For example a differentiable real function is continuous, because the identity function id from $\mathbb{R}$ to $\mathbb{R}$ is continuous. Indeed – when differentiating, we are "comparing" small differences of the type $f(x+h)-f(x)$ and $(x+h)-(x) = id(x+h)-id(x)$ by calculating their quotient. And – in a way – every differentiable function f will "inherit" the continuity of the identity function. Two sufficient conditions, presented in this paper, are based on this idea of comparison.

The classical notion of relative derivative replaces the identity function $id\colon \mathbb{R} \to \mathbb{R}$ by a function $g\colon \mathbb{R} \to \mathbb{R}$ (e.g. in [1] or [8]). In this paper we are going to define a new notion of a generalized relative derivative.

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ON UPPER AND LOWER ALMOST M-ITERATE CONTINUOUS MULTIFUNCTIONS

By
TAKASHI NOIRI AND VALERIU POPA
(Received April 4, 2014)

Abstract. We introduce the notion of *mIT*-structures determined by operators mInt and mCl on an m-space (X, m_X). By using *mIT*-structures, we introduce and investigate a multifunction $F\colon (X, m_X) \to (Y, \sigma)$ called upper/lower almost *mIT*-continuous. As special cases of upper/lower almost *mIT*-continuity, we obtain upper/lower almost γ-M continuity [36] and upper/lower almost δ-M-precontinuity [37].

1. Introduction

Semi-open sets, preopen sets, α-open sets, β-open sets, γ-open sets and δ-open sets play an important role in the researches of generalizations of continuity in topological spaces. By using these sets, several authors introduced and studied various types of weak forms of continuity for functions and multifunctions. In 1968, Singal and Singal [33] introduced the notion of almost continuous functions. In 1982, Popa [24] introduced the concepts of upper/lower almost continuous multifunctions. In [9], [21], [25], [29], [31] and other papers, other forms of almost continuous multifunctions are introduced and investigated.

In [26] and [27], the present authors introduced and studied the notions of minimal structures, m-spaces, m-continuity, M-continuity and other notions. In [28], the notion of almost m-continuous functions is introduced and studied. Recently, in [23], a unified theory of almost continuity for multifunctions is obtained.

Quite recently, in [15], [16], [17], [18], and [19], Min and Kim introduced the notions of m-semi-open sets, m-preopen sets, m-α-open sets and m-β-open sets which generalize the notions of semi-open sets, preopen sets, α-open sets

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ON GENERALIZED SIDON SETS WHICH ARE ASYMPTOTIC BASES

By
SÁNDOR Z. KISS

(Received May 15, 2014)

Abstract. Let h and k be positive integers. We say a set $\mathcal{A}$ of positive integers is an asymptotic basis of order k if every large enough positive integer can be represented as the sum of k terms from $\mathcal{A}$. A set of positive integers $\mathcal{A}$ is called $B_h[g]$ set if all positive integers can be represented as the sum of h terms from $\mathcal{A}$ at most g times. In this paper we prove the existence of $B_h[g]$ sets which are asymptotic bases of order k, if $3 \leq h < k$ by using probabilistic methods.

1. Introduction

Let $\mathbb{N}$ denote the set of positive integers. Let h and k be positive integers satisfying $3 \leq h < k$. Let $\mathcal{A} \subset \mathbb{N}$ be an infinite set of positive integers and let $R_h(\mathcal{A}, n)$ denote the number of solutions of the equation

$$(1) \quad a_1 + a_2 + \cdots + a_h = n, \quad a_1 \in \mathcal{A}, \ldots, a_h \in \mathcal{A}, \quad a_1 \leq a_2 \leq \ldots \leq a_h,$$

where $n \in \mathbb{N}$. A set of positive integers $\mathcal{A}$ is called $B_h[g]$ set if for every $n \in \mathbb{N}$, the number of representations of n as the sum of h terms in the form (1) is at most g, that is $R_h(\mathcal{A}, n) \leq g$. We say a set $\mathcal{A} \subset \mathbb{N}$ is an asymptotic basis of order k, if $R_k(\mathcal{A}, n) > 0$ for all large enough positive integer n, i.e., if there exists a positive integer n_0 such that $R_k(\mathcal{A}, n) > 0$ for $n > n_0$. In [5] and [6] P. Erdős, A. Sárközy and V. T. Sós asked if there exists a Sidon set (or $B_2[1]$ set) which is an asymptotic basis of order 3. The problem also appears in [13] (with a typo in it: order 2 is written instead of order 3). It is easy to see that a Sidon set cannot be an asymptotic basis of order 2 (see in [8]). Recently J. M. Deshouillers and A. Plagne in [2] constructed a Sidon set which is an asymptotic basis of order at

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