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SECTIO GEOGRAPHICA
incepit anno MCMLXVI
SECTIO GEOLOGICA
incepit anno MCMLVII
SECTIO GEOPHYSICA ET METEOROLOGICA
incepit anno MCMLXXV
SECTIO HISTORICA
incepit anno MCMLVII
SECTIO IURIDICA
incepit anno MCMLIX
SECTIO LINGUISTICA
incepit anno MCMLXX
SECTIO MATHEMATICA
incepit anno MCMLVIII
SECTIO PAEDAGOGICA ET PSYCHOLOGICA
incepit anno MCMLXX
SECTIO PHILOLOGICA
incepit anno MCMLVII
SECTIO PHILOLOGICA HUNGARICA
incepit anno MCMLXX
SECTIO PHILOLOGICA MODERNA
incepit anno MCMLXX
SECTIO PHILOSOPHICA ET SOCIOLOGICA
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ANNALES UNIV. SCI. BUDAPEST., **56** (2013), 3–37

THE ZARANKIEWICZ PROBLEM, CAGES, AND GEOMETRIES

By
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Dedicated to the memories of András Gács and István Reiman.

Abstract. In the paper we consider some constructions of $(k, 6)$-graphs that are isomorphic to an induced subgraph of the incidence graph of a finite projective plane, and present some unifying concepts. Also, we obtain new bounds on and exact values of Zarankiewicz numbers, mainly when the parameters are close to those of a design.

1. Introduction

This paper is dedicated to the memory of András Gács and István Reiman. We wish to present results on two well-known extremal graph theoretic problems, (k, g)-graphs (related to cages) and the Zarankiewicz problem, that András worked on in the last period of his life. These topics in some cases have close relations to finite geometry, and design theory. The first, pioneering results in exploring these connections are due to István Reiman [37, 38] in case of the Zarankiewicz problem. Although we formulate some results in more general settings, we mainly focus on issues that are related to finite projective planes.

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TOTALLY (μ, λ)-CONTINUOUS AND SLIGHTLY (μ, λ)-CONTINUOUS FUNCTIONS IN GENERALIZED TOPOLOGICAL SPACES

By
JAMAL M. MUSTAFA

(Received December 2, 2011)

Abstract. In this paper, totally (μ, λ)-continuity and slightly (μ, λ)-continuity are introduced and studied. Furthermore, basic properties and preservation theorems of totally(μ, λ)-continuous and slightly (μ, λ)-continuous functions are investigated and the relationships between these functions and their relationships with some other functions are investigated.

1. Introduction and preliminaries

In [1]–[12], Á. Császár founded the theory of generalized topological spaces, and studied the elementary character of these classes. Especially he introduced the notions of continuous functions on generalized topological spaces, and investigated characterizations of generalized continuous functions (= (μ, λ)-continuous functions in [3]). We recall some notions defined in [3]. Let X be a non-empty set and $\exp X$ the power set of X. We call a class $\mu \subseteq \exp X$ a generalized topology [3] if $\phi \in \mu$ and the arbitrary union of elements of μ belongs to μ. A set X with a generalized topology μ on it is called a generalized topological space and is denoted by (X, μ).

For a generalized topological space (X, μ), the elements of μ are called μ-open sets and the complements of μ-open sets are called μ-closed sets. For $A \subseteq X$, we denote by $c_\mu(A)$ the intersection of all μ-closed sets containing A, i.e., the smallest μ-closed set containing A; and by $i_\mu(A)$ the union of all μ-open

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$\wedge_w$-SETS AND $\vee_w$-SETS IN WEAK STRUCTURES

By
AHMAD AL-OMARI AND TAKASHI NOIRI

(Received December 22, 2011)

Abstract. In this paper we introduce the concepts of $\wedge_w$-sets and $\vee_w$-sets in a weak structure space due to Császár. It is shown that many results in previous papers can be considered as special cases of our results.

1. Introduction

The notion of $\wedge$-sets was introduced by Maki [5] in 1986. A subset A of a topological space is called a $\wedge$-set if it is the intersection of all open sets containing A. Recently many authors have introduced and studied modifications of $\wedge$-sets. By using a minimal structure, Cammaroto and Noiri [1] introduced the notions of $\wedge_m$-sets and $\vee_m$-sets as unified forms of these modifications. Furthermore, recently Ekici and Roy [4] have introduced and investigated the notions of $\wedge_\mu$-sets and $\vee_\mu$-sets on a generalized topological space (X, μ) due to Császár [2]. Quite recently, Császár [3] has introduced the notion of weak structures and obtained several fundamental properties of weak structures.

In this paper, we introduce the notions of $\wedge_w$-sets and $\vee_w$-sets on a weak structure space (X, w) and investigate the properties of sets and spaces related to $\wedge_w$-sets and $\vee_w$-sets.

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SUPRA β-OPEN SETS AND SUPRA β-CONTINUITY ON TOPOLOGICAL SPACES

By
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(Received December 23, 2011)

Abstract. In this paper, a new class of sets and maps between topological spaces called supra β-open sets and supra β-continuous maps, respectively are introduced and studied. Furthermore, the concepts of supra β-open maps and supra β-closed maps in terms of supra β-open sets and supra β-closed sets, respectively, are introduced and several properties are investigated.

1. Introduction and preliminaries

The concept of supra topology is fundamental with respect to the investigation of general topological spaces. Extensive research was done by many mathematicians in supra topology [1, 2, 3, 4]. They generalized the concept of openness such as supra-open, supra α-open, supra-preopen, supra b-openness and obtained many important results analogous to topological spaces. In 1983, Mashhour et al. [1] initiated the study of the so-called supra topological spaces and studied S-continuous maps and S^*-continuous maps. We will use the term supra-continuous maps instead of S-continuous maps.

In 2008, Devi et al. [2] introduced and studied a class of sets and maps between topological spaces called supra α-open sets and supra α-continuous maps, respectively. Recently, Sayed and Noiri [3] introduced and investigated the notions of supra b-continuity, supra b-openness and supra b-closedness in terms of supra b-open set and supra b-closed set, respectively and most recently Sayed [4] introduced and investigated the notion of supra-pre-continuity, supra-pre openness and supra-pre closedness sets in terms of supra pre-open set and

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ABSOLUTE CONVERGENCE OF WALSH-FOURIER SERIES

By
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Abstract. For the classes of functions of $\Lambda BV(p(n) \uparrow \infty, \varphi)$ and $BV \cap Lip(\alpha, p)$, we obtain sufficiency conditions for the convergent of series $\sum_{n=1}^{\infty} n^{\alpha} |\hat{f}(n)|^{\beta}$, ($\alpha \geq 0$, $0 < \beta \leq 2$), where $\hat{f}(n)$ are Walsh-Fourier coefficients of f.

1. Introduction

In 1949, N. J. Fine [1] estimated the order of magnitude of Walsh-Fourier coefficients of a function satisfies Lipschitz condition of order α, $0 < \alpha \leq 1$. In 2001, U. Goginava [2] obtained sufficiency condition for the uniform convergence of Walsh-Fourier series of functions of the generalized Wiener class $BV(p(n) \uparrow \infty)$. Peter Simon ([4], [5]) has studied summability of Walsh-Fourier series. Recently, Móricz [3] obtained sufficiency condition for the absolute convergence of Walsh-Fourier series. Here, we obtain sufficiency conditions for the generalized β-absolute convergence of Walsh-Fourier series of classes functions of $\Lambda BV(p(n) \uparrow \infty, \varphi)$ and $BV \cap Lip(\alpha, p)$.

Let f be a function defined on $(-\infty, \infty)$ with period 1. $\mathbf{P}$ is said to be a partition with period 1 if

$$\mathbf{P}: \ \ldots < x_{-1} < x_0 < x_1 < \ldots < x_m < \ldots$$

satisfies $x_{k+m} = x_k + 1$ for $k = 0, \pm 1, \pm 2, \ldots$, where m is a positive integer.

Definition 1.1. Let $\varphi(n)$ be a real sequence such that $\varphi(1) \geq 2$ and $\lim_{n\to\infty} \varphi(n) = \infty$. For a sequence $\Lambda = \{\lambda_m\}$ ($m = 1, 2, \ldots$) of non-decreasing

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A NOTE ON SEPARATION PROPERTIES OF θ-MODIFICATIONS OF TOPOLOGIES

By
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Abstract. In this note, we construct a Urysohn topology such that its θ-modification $\theta(\tau)$ is not T_2, which answers a question in [1] (Problem 2.10 in [1]).

1. Introduction

By a space, we mean a topological space. For a subset A in a space (X, τ), we denote by $int(A)$ and $cl(A)$ for the interior and the closure of A, respectively. A point x of a space X is called a *θ-cluster point* [1] (also called *θ-adherent point* in [5]) of a subset $A \subseteq X$ iff $cl(U) \cap A \neq \emptyset$ whenever U is an open neighbourhood of x. Let $\gamma_\theta(A)$ denote the set of all θ-cluster points of A; A is is called *θ-closed* iff $A = \gamma_\theta A$. A subset U is said to be *θ-open* if its complement is θ-closed. Clearly, a subset U of X is θ-open iff for each point $x \in U$ there exists an open set V containing x such that $V \subseteq cl(V) \subseteq U$.

The collection of all θ-open sets forms a topology $\theta(\tau)$ on X. This topology is coarser than τ and called the *θ-modification* [1] of the topology τ. A topology τ is said to be *Urysohn* [1] iff $x, y \in X$ imply the existence of open sets V and W such that $x \in V$, $y \in W$ and $cl(V) \cap cl(W) = \emptyset$.

In [1], Á. Császár examined the relation of separation properties of τ and its modification $\theta(\tau)$. It has been proved that if $\theta(\tau)$ is T_2 then τ is Uryshon (see Theorem 2.6 in [1]). It is an open question whether the converse is true:

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BETWEEN δ-CLOSED SETS AND δ-g-CLOSED SETS

By
TAKASHI NOIRI AND VALERIU POPA

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Abstract. Quite recently, by using b-open sets [2], Nagaveni and Narmadha [11] have introduced and investigated the notion of rb-closed sets in a topological space. These subsets place between δ-closed sets and δ-g-closed sets due to Dontchev and Ganster [3]. In this paper, we introduce the notion of $m\delta g$-closed sets and obtain the unified theory for certain collections of subsets between δ-closed sets and δ-g-closed sets.

1. Introduction

In 1970, Levine [7] introduced the notion of generalized closed (g-closed) sets in topological spaces. Since then, many variations of g-closed sets are introduced and investigated. Dontchev and Ganster [3] introduced the notions of δ-g-closed sets and $T_{3/4}$-spaces. They showed that the digital line (Z, κ) [5] is a $T_{3/4}$-space but it is not T_1. Quite recently, Nagaveni and Narmadha [11] have introduced the notion of rb-closed sets by using b-open sets and studied their basic properties and characterizations.

In this paper, we introduce the notion of $m\delta g$-closed sets in order to establish the unified theory for certain collections of subsets between δ-closed sets and δ-g-closed sets. And we obtain the basic properties and characterizations of $m\delta g$-closed sets. In the last section, we define several new subsets which lie between δ-closed sets and δ-g-closed sets.

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FREE WEAK LATTICES

By
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1. Introduction

There are many generalizations of the varieties of lattices, as weakly associative lettices (see e.g. [1], [2] and weak lattices [3]). The graphs in [2] have the property, that every pair of distinct elements have unique common upper and lower bounds (UBP). Any algebra satisfying UBP is subdirectly irreducible. (For comparable elements this means that the graph does not contains three-element chains. If a weak lattice satisfies UBP, then it is a weakly associative lattice. However, there are other weak lattices containing no three-element chain; for example the free weak lattices have this property.) UBP were strongly connected to projective planes [4]. The simplest one, having more than two elements is the triangle: $a \to b \to c \to a$. These were **real** generalizations of the two-element lattice. It is still an open question, whether finite ones always contain a triangle. It was given an infinite one in [5] such that subdirectly irreducible members of the variety generated by this algebra having more then two elements contain no triangle. A generalization of these graphs are the **Dual discriminator algebras**, see e.g. in [6].

In [2] free weakly associative lattices were constructed. Here we shall construct free weak lattices.

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GRAPH POLYNOMIALS AND GRAPH TRANSFORMATIONS IN ALGEBRAIC GRAPH THEORY
Abstract of Ph.D. Thesis

By
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(Defended November 11, 2011)

1. Introduction

The algebraic graph theory has a long history due to its intimate relationship with chemistry and (statistical) physics. In these fields one often describes a system, state or a molecule by an appropriate parameter. Then it gives rise to the purely mathematical problem to determine what the extremal values of this parameter are. In the dissertation we give some general methods to attack these kinds of extremal problems. In the first, bigger half of the thesis we study two graph transformations, the so-called Kelmans transformation, and the generalized tree shift introduced by the author. The Kelmans transformation can be applied to all graphs, while one can apply the generalized tree shift only to trees. The importance of these transformations lies in the fact that surprisingly many natural graph parameters increase (or decrease) along these transformations. This way we gain a considerable amount of information on the extremal values of the studied parameter.

In the second half of the dissertation we study a purely extremal graph theoretic problem, the so-called density Turán problem which, however, turns out to be strongly related to algebraic graph theory in several ways. As a by-product of the efforts we did to solve the problem we give a solution to a longstanding open problem concerning trees having only integer eigenvalues.